Possible Emission of Cosmic $X-$ and $\gamma-$rays by Unstable Particles at Late Times

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Not all astrophysical mechanisms of the emission of electromagnetic radiation including $X-$ and $\gamma-$ rays coming from the space are clear. We find that charged unstable particles as well as neutral unstable particles with non–zero magnetic moment which live sufficiently long may emit electromagnetic radiation. This new mechanism is connected with the properties of unstable particles at the post exponential time region. Analyzing the transition time region between exponential and non-exponential form of the survival amplitude it is found that the instantaneous energy of the unstable particle can take very large values, much larger than the energy of this state for times from the exponential time region. Basing on the results obtained for the model considered, it is shown that this purely quantum mechanical effect may be responsible for causing unstable particles to emit electromagnetic–, $X-$ or $\gamma-$rays at some time intervals from the transition time regions.

Typical physical processes in which cosmic microwave and other electromagnetic radiation, $X-$, or γ-rays are generated have purely electromagnetic nature (an acceleration of charged particles, inverse Compoton scattering, etc.), or have the nature of nuclear and particle physics reactions (particle–antiparticle annihilation, nuclear fusion and fission, nuclear or particle decay). The knowledge of these processes is not sufficient for explaining all mechanisms driving the emission from some galactic and extragalactic $X-$ or $\gamma-$rays sources, e.g. the mechanism that generates $\gamma-$ray emission of the so-called "Fermi bubbles" remains controversial, the mechanisms which drive the high energy emission from blazars is still poorly understood, etc. (see eg. [1–3]). Astrophysical processes are the source of not only electromagnetic, X- or γ-rays but also a huge number of elementary particles including unstable particles of very high energies (see eg. [3]). The numbers of created unstable particles during these processes are so large that some of them can survive up to times t at which the survival probability depending on t transforms from the exponential form into the inverse power–like form. It appears that at this time region a new quantum effect is observed: A very rapid fluctuations of the instantaneous energy of unstable particles take place. These fluctuations of the instantaneous energy should manifest itself as fluctuations of the velocity of the particle. We show that this effect may cause unstable particles to emit electromagnetic radiation of a very wide spectrum: from radio– up to ultra–high frequencies ν including $X-$rays and $\gamma-$rays.

Let us start from a brief introduction into the problem of the late time behavior of unstable states. If $|\phi\rangle$ is an initial unstable state then the survival probability, $\mathcal{P}(t)$, equals $\mathcal{P}(t) = |a(t)|^2$, where $a(t)$ is the survival amplitude, $a(t) = \langle\phi|\phi;t\rangle$, and $|\phi;t\rangle = e^{-itH}\,|\phi\rangle$, H is the total Hamiltonian of the system under considerations. The spectrum, $\sigma(H)$, of H is assumed to be bounded from below, $\sigma(H) = [E_{min}, \infty)$ and $E_{min} > -\infty$. Searching for late time properties of unstable states one usually uses the integral representation of $a(t)$ as the Fourier transform of the energy distribution function, $\omega(E)$,

$$a(t) = \int \omega(E)\, e^{-itE}\, dE, \tag{1}$$

with $\omega(E) \geq 0$ and $\omega(E) = 0$ for $E < E_{min}$ [4–13]. In the case of quasi–stationary (metastable) states it is convenient to express $a(t)$ in the following form [11, 12], $a(t) = a_{exp}(t) + a_{lt}(t)$, where $a_{exp}(t)$ is the exponential part of $a(t)$, that is $a_{exp}(t) = N\,\exp\,[-it(E_{\phi}^0 - \frac{i}{2}\,\varGamma_{\phi}^0)]$, ($E_{\phi}^0$ is the energy of the system in the state $|\phi\rangle$ measured at the canonical decay times, i.e. when $\mathcal{P}_{\phi}(t)$ has the exponential form, $\varGamma_{\phi}^0$ is the decay width, N is the normalization constant), and $a_{lt}(t)$ is the late time non–exponential part of $a(t)$.

From the literature it is known that the characteristic feature of survival probabilities $\mathcal{P}(t)$ is the presence of sharp and frequent fluctuations at the transition times region, when contributions from $|a_{exp}(t)|^2$ and $|a_{lt}(t)|^2$ into $\mathcal{P}(t)$ are comparable (see, eg. [5, 7–10]), and that the amplitude $a_{lt}(t)$ and thus the probability $\mathcal{P}(t)$ exhibits inverse power–law behavior at the late time region for times t much later than the crossover time T. (This effect was confirmed experimentally not long ago [14]). The crossover time T can be found by solving the following equation, $|a_{exp}(t)|^2 = |a_{lt}(t)|^2$. In general $T \gg \tau_{\phi}$, where $\tau_{\phi} = 1/\varGamma_{\phi}$ is the live–time of ϕ. Formulae for T depend on the model considered (i.e. on $\omega(E)$) in general (see, eg. [6–8, 11–13]).

It is commonly known that the information about the decay rate, $\varGamma_{\phi}$, of the unstable state $|\phi\rangle$ under considerations can be extracted from the survival amplitude $a(t)$. In general not only $\varGamma_{\phi}$ but also the instantaneous energy $\mathcal{E}_{\phi}(t)$ of an unstable state $|\phi\rangle$ can be calculated using $a(t)$ [11, 12]. In the considered case, $\mathcal{E}_{\phi}(t)$ can be found using the effective Hamiltonian, $h_{\phi}(t)$, governing the time evolution in an one–dimensional subspace of states spanned by vector $|\phi\rangle$, [11, 12]:

$$h_{\phi}(t) = \frac{i}{a(t)}\,\frac{\partial a(t)}{\partial t} \equiv \frac{\langle\phi|H|\phi;t\rangle}{\langle\phi|\phi;t\rangle}. \tag{2}$$

The instantaneous energy $\mathcal{E}_\phi(t)$ of the system in the state $|\phi\rangle$ is the real part of $h_\phi(t)$, $\mathcal{E}_\phi(t) = \Re\,(h_\phi(t))$. The imaginary part of $h_\phi(t)$ defines the instantaneous decay rate $\varGamma_\phi(t)$, $\varGamma_\phi \equiv \varGamma_\phi(t) = -2\,\Im\,(h_\phi(t))$.

There is $\mathcal{E}_\phi(t) = E_\phi^0$ and $\varGamma_\phi(t) = \varGamma_\phi^{\,0}$ at the canonical decay times and at asymptotically late times,

$$\mathcal{E}_\phi(t) \simeq E_{min} + \frac{c_2}{t^2} + \frac{c_4}{t^4}\ldots, \quad (\text{for } \ t \gg T), \tag{3}$$

$$\varGamma_\phi(t) \simeq \frac{c_1}{t} + \frac{c_3}{t^3} + \ldots \quad (\text{for } \ t \gg T), \tag{4}$$

where $c_i = c_i^*,\ i = 1, 2, \ldots$, so $\lim_{t\to\infty} \mathcal{E}_\phi(t) = E_{min}$ and $\lim_{t\to\infty} \varGamma_\phi(t) = 0$ [11, 12, 15]. Results (3) and (4) are rigorous. The basic physical factor forcing the amplitude $a(t)$ to exhibit inverse power law behavior at $t \gg T$ is the boundedness from below of $\sigma(H)$. This means that if this condition is satisfied and $\int_{-\infty}^{+\infty} \omega(E)\,dE < \infty$, then all properties of $a(t)$, including the form of the time–dependence at $t \gg T$, are the mathematical consequence of them both. The same applies by (2) to the properties of $h_\phi(t)$ and concerns the asymptotic form of $h_\phi(t)$ and thus of $\mathcal{E}_\phi(t)$ and $\varGamma_\phi(t)$ at $t \gg T$.

The sharp and frequent of fluctuations of $\mathcal{P}_\phi(t)$ at the transition times region are a consequence of a similar behavior of real and imaginary parts of the amplitude $a(t)$ at this time region. Therefore the derivatives of $a(t)$ may reach extremely large negative and positive values for some times from the transition time region and the modulus of these derivatives is much larger than the modulus of $a(t)$, which is very small for these times. This means that at this time region the real part of $h_\phi(t)$ which is expressed by the relation (2), ie. by a large derivative of $a(t)$ divided by a very small $a(t)$, can reach values much larger than the energy E_ϕ^0 of the unstable state measured at the canonical decay times. Using relations (1), (2) and assuming the form of $\omega(E)$ and performing all necessary calculations numerically one can see how this mechanism work. A typical behavior of the instantaneous energy $\mathcal{E}_\phi(t)$ at the transition time region is presented in Figs (1) and (2). In these figures the calculations were performed for $E_{min} = 0$ and the Breit–Wigner energy distribution function, $\omega(E) \equiv \omega_{BW}(E) \stackrel{\text{def}}{=} \frac{N}{2\pi}\,\varTheta(E)\,\frac{\varGamma_\phi^{\,0}}{(E-E_\phi^0)^2+(\varGamma_\phi^{\,0}/2)^2}$, where $\varTheta(E)$ is the unit step function.

Note that from the point of view of a frame of reference in which the time evolution of the unstable system was calculated the Rothe experiment as well as the picture presented in Figs (1), (2) refer to the rest coordinate system of the unstable system considered. Astrophysical sources of unstable particles emit them with relativistic or ultra–relativistic velocities in relation to an external observer so many of these particles move in space with ultra high energies. The question is what effects can be observed by an external observer when the unstable particle, say ϕ, which survived up to the transition times region, $t \sim T$, or longer is moving with a relativistic velocity in relation to this observer. The distance d from the

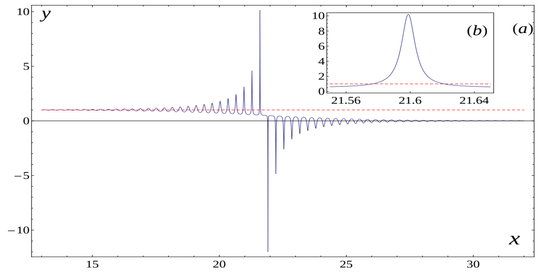

Figure 1. (a) The instantaneous energy $\mathcal{E}_\phi(t)$ in the transitions time region: The case $E_\phi^0/\varGamma_\phi^{\,0} = 20$. Axes: $y = \mathcal{E}_\phi(t)/E_\phi^0$, $x = t/\tau_\phi$. The dashed line denotes the straight line $y = 1$. (b) Enlarged part of (a): The highest maximum of $\mathcal{E}_\phi(t)/E_\phi^0$ in the transition times region.

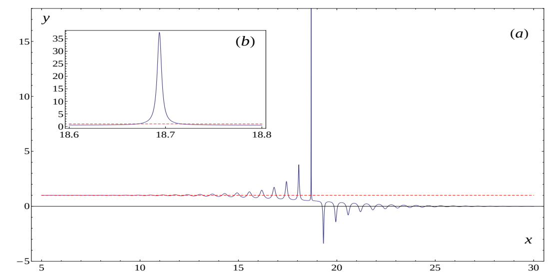

Figure 2. The same as in Fig (1) for $E_\phi^0/\varGamma_\phi^{\,0} = 10$.

source reached by this particle is of order $d \sim d_T$, where $d_T = v^\phi \cdot T'$, $T' = \gamma_L\,T$ and $\gamma_L \equiv \gamma_L(v^\phi) = (\sqrt{1-\beta^2})^{-1}$, $\beta = v^\phi/c$, v^ϕ is the velocity of the particle ϕ. (For simplicity we assume that there is a frame of reference common for the source and observer both and that they do not move with respect to this frame of reference). The last expression in relation (2) explains why effects of type (3), (4) and those one can see in Figs (1), (2) are possible. In the case of moving particles created in astrophysical processes one should consider the effect shown in Figs (1), (2) together with the fact that the particle gains extremely huge kinetic energy, W^ϕ, which have to be conserved. There is $W^\phi = m_\phi^0\ c^2\ \gamma_L$, where m_ϕ^0 is the rest mass of the particle ϕ. We have $m_\phi^0\ c^2 \equiv E_\phi^0$ at canonical decay times and thus $W^\phi \equiv E_\phi^0\ \gamma_L$ at these times. At this time region $E_\phi^0 = \mathcal{E}_\phi(t)$ but at times $t \gg \tau_\phi$, $t \sim T$ we have $\mathcal{E}_\phi(t) \neq E_\phi^0$. So at transition times region the relation $\mathcal{E}_\phi(t) = m_\phi^0(t)\,c^2$ defines the instantaneous rest mass $m_\phi^0(t)$ and the kinetic energy of the particle ϕ at $t \sim T$ equals

$$W^\phi(t) = m_\phi^0(t)\,c^2\,\gamma_L(t) \equiv \mathcal{E}_\phi(t)\,\gamma_L(t), \quad (\text{for } t \sim T). \tag{5}$$

Here $\gamma_L(t) = \gamma_L(v^\phi(t))$, $v^\phi(t)$ is the velocity of the particle ϕ at the instant t. Of course the kinetic energies $W^\phi, W^\phi(t)$ of ϕ have to be the same at the canonical decay times region and at times $t \sim T$: $W^\phi \equiv W^\phi(t)$, that is there should be

$$W^\phi \equiv \mathcal{E}_\phi(t)\,\gamma_L(t)\ =\ const. \tag{6}$$

From relation (6) one can infer that this is possible only when the changes of $\mathcal{E}_\phi(t)$ at times $t \sim T$ are balanced with suitable changes of $\gamma_L(t)$ (i.e. of the velocity $v^\phi(t)$ of the considered particle). So, in the case of moving unstable particles, an external observer should detect rapid fluctuations (changes) of their velocities at distances $d \sim d_T$ from their source. These fluctuations of the velocities mean for the observer that the particles are moving with a nonzero acceleration in this space region, $\dot{v^\phi} \neq 0$. So we can expect that this observer will register electromagnetic radiation emitted by charged unstable particles, which survived up to times $t \sim T$, i.e. which reached distances $d \sim d_T$ from the source (see Fig (3)). This follows from the Larmor formula and its relativistic generalization, which state that the total radiation power, P, from the considered charged particle is proportional to $(\dot{v^\phi})^2$, $P \propto (\dot{v^\phi})^2$ (see e.g. [16]) and $\dot{v^\phi} \neq 0$ implies that there must be $P \neq 0$. The same conclusion also concerns neutral unstable particles with non–zero magnetic moment [16, 17]. One should expect that the spectrum of this radiation will be very wide: From high radio frequencies, through X–rays up to high energy γ–rays depending on the scale of the fluctuations of the instantaneous energy $\mathcal{E}_\phi(t)$ in this space region.

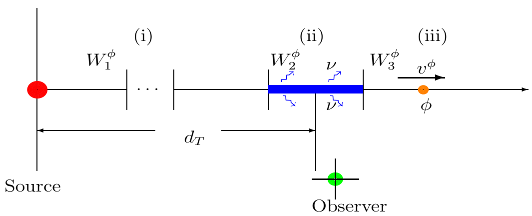

Figure 3. Time regions: (i) Canonical decay, (ii) Transition, (iii) Asymptotically late. $W_i^\phi = W^\phi(t_i)$, $(i = 1, 2, 3)$ and $W_i^\phi(t_i)$ is the energy of moving relativistic particle ϕ, $t_1 \ll t_2 \ll t_3$, $t_1 \sim \tau_\phi$, $t_2 \sim T$, $t_3 \gg T$ and $W^\phi(t_1) = m_\phi^0\, c^2\, \gamma_L = E_\phi^0 \gamma_L$, ν is the frequency of the emitted electromagnetic rays.

Within the model defined by $\omega_{BW}(E)$ the cross–over time T can be found using the following approximate relation valid for $E_\phi^0/\mathit{\Gamma}_\phi^0 \gg 1$, [11]:

$$\mathit{\Gamma}_\phi^0\, T \equiv \frac{T}{\tau_\phi} \;\sim\; 2\,\ln\Big[2\pi\Big(\frac{E_\phi^0}{\mathit{\Gamma}_\phi^0}\Big)^2\Big], \tag{7}$$

whereas for the model considered in [6, 13] one has $T/\tau_\phi \;\sim\; 5\,\ln\big(E_\phi^0/\mathit{\Gamma}_\phi^0\big)$ (see (11) in [13]). Considering a meson $\mu^\pm$ as an example and using (7) one finds $T = T_\mu \sim 170\,\tau_\mu$. (The formula (11) from [13] gives $T_\mu \sim 202\,\tau_\mu$). The distance d_{T_μ} from the source reached by muon, which survived up to the time $T_\mu \simeq 170\,\tau_\mu$ depends on its kinetic energy W^μ and equals: from $d_{T_\mu} \simeq 10^6$ [m] if $W^\mu = 10^9$ [eV], up to $d_{T_\mu} \simeq 0,027$ [pc] if $W^\mu = 10^{18}$ [eV]. Similarly, for π–mesons one obtains: $d_{T_\pi} \simeq 8.5 \times 10^3$ [m] if $W^\pi = 10^9$ [eV] and $d_{T_\pi} \simeq 8.6 \times 10^{12}$ [m] if $W^\pi = 10^{18}$ [eV]. For the neutron one finds: $d_{T_n} \simeq 16,23$ [au] if $W^n = 10^9$ [eV] and $d_{T_n} \simeq 2228$ [kpc] if $W^n = 10^{18}$ [eV].

Let us now analyze Fig (1) in more details. Coordinates of the highest maximum in Fig (1) are equal: $(x_{mx}, y_{mx}) = (21.60, 10.27)$. Coordinates of points of the intersection of this maksimum with the straight line $y = 1$ are equal: $(x_1, y_1) = (21.58, 1.0)$ and $(x_2, y_2) = (21.62, 1.0)$. From these coordinates one can extract the change $\Delta v^\phi = v^\phi(t_{mx}) - v^\phi(t_1)$ of the velocity v^ϕ of the considered particle and the time interval $\Delta t = t_{mx} - t_1$ at which this change occurred. Indeed, using (6) one finds

$$\gamma_L(t_1) = \frac{\mathcal{E}_\phi(t_{mx})}{\mathcal{E}_\phi(t_1)}\,\gamma_L(t_{mx}). \tag{8}$$

There is $\mathcal{E}_\phi(t_1) \equiv E_\phi^0$ in the considered case. This means that we can replace $\gamma_L(t_1)$ by γ_L measured at the canonical decay times and then taking the value of the ratio $\mathcal{E}_\phi(t_{mx})/E_\phi^0$ from Fig (1) we can use (8) to calculate $\gamma_L(t_{mx})$. Next having $\gamma_L(t_1) \equiv \gamma_L$ and $\gamma_L(t_{mx})$ it is easy to find $\Delta v^\phi = v^\phi(t_{mx}) - v^\phi(t_1)$. Now using relativistic generalization of the Larmor formula (see eg. formula (14.43) in [16]) one can estimate the energy P of the electromagnetic radiation emitted in unit of time by an unstable charged relativistic particle ϕ during the time interval Δt: That is one can find $\Delta v^\phi/\Delta t$ and thus $P \propto (\Delta v^\phi/\Delta t)^2$. This procedure, formulae (6), (8) and parameters describing the highest maximum in Fig. (1) lead to the following (simplified, very conservative) estimations of the energies of the electromagnetic radiation emitted by ultra relativistic muon at the transition times region (in a distance $d \sim d_T$ from the source): $P \sim 4.6$ [eV/s]. Analogously coordinates of the highest maximum in Fig (2) are equal: $(x_{mx}, y_{mx}) = (18.69, 37.68)$ and coordinates of points of the intersection of this maksimum with the line $y = 1$ are: $(x_1, y_1) = (18.67, 1.0)$ and $(x_2, y_2) = (18.72, 1.0)$. This leads to the following estimation: $P \sim 0.84$ [keV/s]. Similar estimations of P can be found for neutral ultra–relativistic unstable particles with non–zero magnetic moment.

The question is where the above described effect may be observed. Astrophysical and cosmological processes in which extremely huge numbers of unstable particles are created seem to be a possibility for the above discussed effect to become manifest. The fact is that the probability $\mathcal{P}_\phi(t) = |a(t)|^2$ that an unstable particle ϕ survives up to time $t \sim T$ is extremely small. Let $\mathcal{P}_\phi(t)$ be $\mathcal{P}_\phi(t)\big|_{t \sim T} \;\sim\; 10^{-k}$, where $k \gg 1$, then there is a chance to observe some of particles ϕ survived at $t \sim T$ only if there is a source creating these particles in $\mathcal{N}_\phi$ number such that $\mathcal{P}_\phi(t)\big|_{t\sim T}\,\mathcal{N}_\phi \;\gg\; 1$. So if a source exists that creates a flux containing $\mathcal{N}_\phi \;\sim\; 10^{\,l}$, unstable particles and $l \gg k$ then the probability theory states that the number N_{surv} of unstable particles $N_{surv} = \mathcal{P}_\phi(t)\big|_{t\sim T}\,\mathcal{N}_\phi \;\sim\; 10^{l-k} \;\gg\; 1$, has to survive up to time $t \sim T$. Sources creating such numbers of unstable particles are known from cosmology and astrophysics: as

example of such a source can be considered processes taking place in galactic nuclei (galactic cores), inside stars, etc. According to estimations of the luminosity of some γ–rays sources the energy emitted by these sources can even reach a value of order 10^{52} [erg/s], [3, 18–20], and it is only a part of the total energy produced there. So, if one has a source emitting energy 10^{50} [erg/s] then, eg., an emission of $\mathcal{N}_0 \simeq 6.25 \times 10^{47}$ [1/s] particles of energy 10^{18} [eV] is energetically allowed. The same source can emit $\mathcal{N}_0 \simeq 6.25 \times 10^{56}$ [1/s] particles of energy 10^9 [eV] and so on. If one follows [13] and assumes that for laboratory systems a typical value of the ratio $E_\phi^0/\mathit{\Gamma}_\phi^0$ is $E_\phi^0/\mathit{\Gamma}_\phi^0 \geq O(10^3 - 10^6)$ and then taking, eg. $E_\phi^0/\mathit{\Gamma}_\phi^0 = 10^6$ one obtains from (7) that $\mathcal{N}_\phi(T) \sim 2.5 \times 10^{-26}\mathcal{N}_0$ and from the estimation of T used in [13] (see (11), (12) in [13]) that $\mathcal{N}_\phi(T) \sim 10^{-30}\mathcal{N}_0$. This means that there are $\mathcal{N}_\phi(T) \sim 14 \times 10^{21}$ particles per second of energy $W^\phi = 10^{18}$ [eV] or $\mathcal{N}_\phi(T) \sim 14 \times 10^{30}$ particles of energy $W^\phi = 10^9$ [eV] in the case of the considered example and T calculated using (7). On the other hand from T obtained for the model considered in [13] one finds $\mathcal{N}_\phi(T) \sim 6.25 \times 10^{17}$ and $\mathcal{N}_\phi(T) \sim 6.25 \times 10^{26}$ respectively. These estimations show that astrophysical sources are able to create such numbers $\mathcal{N}_0$ of unstable particles that sufficiently large number $\mathcal{N}_\phi(T) \gg 1$ of them has to survive up to times T when the effect described above should occur. So the numbers of unstable particles produced by some astrophysical sources are sufficiently large in order that a significant part of them had to survive up to the transition times and therefore to emit electromagnetic radiation. The expected spectrum of this radiation can be very wide: From radio frequencies up to γ–rays depending on energy distribution function $\omega(E)$ of the unstable particle emitting this radiation.

We have shown that charged unstable particles or neutral unstable particles with non–zero magnetic moment, which survived up to transition times or longer, should emit electromagnetic radiation. We have also shown that only astrophysical processes can generate sufficiently huge number of unstable particles in order that this emission could occur. From our analysis it seems to be clear that the effect described in this letter may have an astrophysical meaning and help explain the controversies, which still remain, concerning the mechanisms that generates the cosmic microwave, or X–, or γ–rays emission, e.g. it could help explain why some space areas (bubbles) without visible astronomical objects emit microwave radiation, X– or γ–rays. Indeed, let us consider active galactic nuclei as an example. They emit extremely huge numbers of stable and unstable particles including neutrons (see eg. [2]) along the axis of rotation of the galaxy. The unstable particles, which reached distances $d \sim d_T$ from the galactic plane, should emit electromagnetic radiation. So a distant observer should detect enhanced emission of this radiation coming from bubbles with the centra located on the axis of the galactic rotation at average distances d_T from the galactic plane (see Fig. (3)). In the case of neutrons d_{T_n} can be extremely large. Therefore a possible emission of the electromagnetic radiation generated by neutrons surviving sufficiently long seems to be relatively easy to observe and it should be possible to determine d_{T_n}. Now having realistic sufficiently accurate $\omega(E)$ for neutrons we are able to calculate T_n and to find $\mathcal{E}_n(t)$ and its local maxima at transition times. Thus if the energies W^n, (i.e., γ_L), are known then in fact we know velocities v^n and we can compute d_{T_n} and distances where $\mathcal{E}_n(t)$ has maxima. All these distances fix the space areas where the mechanism discussed should manifest itself. This suggests how to test this mechanism: The computed d_{T_n} can be compared with observational data and thus one can test if the mechanism described in our letter works in astrophysical processes.

Note that all possible effects discussed in this paper are the simple consequence of the fact that the instantaneous energy $\mathcal{E}_\phi(t)$ of unstable particles becomes large for suitably long times compared with $\mathcal{E}_\phi^0$ and for some times even extremely large. This property of $\mathcal{E}_\phi(t)$ is a purely quantum effect resulting from the assumption that the energy spectrum is bounded from below and it was found by performing an analysis of the properties of the quantum mechanical survival probability $a(t)$.

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